Lie ring methods in group theory

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Timetable: 5 lectures (50 minutes each lecture), Escola de Álgebra 2026, 29/06 - 04/07 2026

Aim: The minicourse provides an introduction to a Lie ring method of studying groups, which is based on the so-called associated Lie rings, and aims at illustrating how a Lie theoretic result of E. Zelmanov enables one to treat problems in group theory.

Course contents/ Tentative lectures schedule:

•(Lecture 1) Motivation: the Restricted Burnside Problem - a historical overview.

•(Lecture 2) Lie rings and Lie algebras: definitions and basic properties. Results on Lie ring analogous to theorems about groups (nilpotency, solubility, etc). The technique of extending the ground ring.

•(Lecture 3) Constructing a Lie ring from a group: the associated Lie ring. The result of E. Zelmanov and the solution of the Restricted Burnside Problem.

• (Lecture 4) Applications to Engel groups.

• (Lecture 5) Applications to fixed points of automorphisms.

References

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[2] B. Huppert and N. Blackburn, *Finite Groups II*, Springer-Verlag, Berlin, (1982).

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[4] E. I. Khukhro and P. Shumyatsky, Bounding the exponent of a finite group with automorphisms, *J. Algebra* **212** (1999) 363–374.

[5] P. Shumyatsky, Applications of Lie ring methods to group theory, in *Nonassociative Algebra and Its Applications*, (Eds R. Costa et al.), Marcel Dekker, New York, (2000), 373–395.

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[7] E. I. Zelmanov, Lie methods in the theory of nilpotent groups, in *Groups '93 Galaway/St Andrews*, Cambridge University Press, Cambridge, (1995), 567–585.

[8] E. Zelmanov, Lie algebras and torsion groups with identity, *J. Comb. Algebra* 1, no. 3 (2017), 289–340.